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## Effects of electron–electron scattering on impurity resistivity

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**Abstract.** All important theories of impurity resistivity each give one of two results. These two results coincide for zero temperature but differ for higher temperatures. The electron–electron scattering does not give a direct contribution to the resistivity. However, it has an indirect effect. It affects the occupation numbers of the electron states and thereby modifies the impurity scattering. The two results that we have discussed are both obtained in the absence of explicit electron–electron scattering.

The solution of the Boltzmann equation in absence of electron–electron scattering gives one of the two results. In the limit of strong electron–electron scattering the second result is obtained; strong electron–electron interaction leads to full thermalisation in the centre-of-mass system of the carriers. Thus the two results are the extreme limits of the results from the Boltzmann equation for varying strength of the electron–electron scattering. In the present work we make a realistic estimate of the electron–electron scattering-rate and calculate the Boltzmann result for doped GaAs.

### 1. Introduction

All important theories of impurity resistivity have one of the following two results:

$$\sigma = (ne^2/m^*)\langle\tau(k)\rangle \quad \rho = (m^*/ne^2)1/\langle\tau(k)\rangle \quad (1)$$

and

$$\sigma = (ne^2/m^*)1/\langle1/\tau(k)\rangle \quad \rho = (m^*/ne^2)\langle1/\tau(k)\rangle. \quad (2)$$

The time  $\tau(k)$  in these equations is the transport time. The angle bracket stands for the following averaging procedure:

$$\langle A(k) \rangle = -\frac{4}{3n} \int \frac{d^3k}{(2\pi)^3} \frac{\partial f^0(k)}{\partial \varepsilon_k} \varepsilon_k A(k). \quad (3)$$

Hereafter we shall refer to (1) as result number one or the first result and to (2) as result number two or the second result.

The two results are equal for zero temperature but differ for finite temperatures. The first result is obtained from the solution of the standard semi-classical Boltzmann equation, from the Kubo formula for the current–current correlation function or from the quantum Boltzmann equation (see chapter 7.1 of [1] for the derivation of the results from these three formulations). The second result is obtained from many different

formulations. It is obtained in the force-balance method [2–8]; as the lowest order contribution in the resistivity formulations [9–13]; in the energy-loss method [14]; in the generalized Drude approach (GDA) [15]; as the simplest variational result in the variational solution to the Boltzmann equation [16]; from the Kubo formula for the force-force correlation function [17].

All these derivations have been performed without any explicit inclusion of electron–electron (e–e) scattering. The e–e scattering has no direct effect on the resistivity. However it has an indirect effect. It affects the occupation numbers of the electron states and thereby modifies the impurity scattering. The e–e scattering strives towards thermalisation in the centre-of-mass (CM) system of the carriers. In the energy-loss method one of the basic assumptions is that the carriers are thermalised in CM. For this assumption to be valid one needs very strong e–e scattering. The solution to the Boltzmann equation in the limit of very strong electron–electron interaction [18, 19] gives result number two. Both the resistivity formulations and the force-balance type theories have been shown to be incorrectly solved [20–23]. There are higher order diverging terms that if included and properly summed over will bring the result to the result number one. However, in the presence of inelastic scattering processes, like electron–phonon or e–e scattering, the divergences disappear [18] and the result lies somewhere in between the first and second results.

Thus we can summarise the situation in the following way. If the e–e scattering is very weak, result number one is correct; if the e–e scattering is very strong, result number two is correct; if the e–e scattering-rate is of the same order of magnitude as the electron–impurity scattering-rate the correct result lies somewhere in between results number one and two.

The purpose of his work, which is a short sequel to [19], is to find out how important the e–e scattering is and which of the two results are closest to the correct result for a realistic situation. To do this we determine the strength of the e–e scattering and use the Boltzmann equation with e–e scattering included to calculate numerical results. The calculation is performed for n-type doped GaAs. In section 2 we make a brief derivation of the Boltzmann expression in the presence of finite e–e scattering. The derivation of the e–e scattering-rate  $1/\tau_{e-e}$  is presented in section 3. By using a trick we can obtain this quantity from the GDA. The results, in the form of the reversal mobility, are presented in section 4. Finally, a summary is given in section 5.

## 2. Boltzmann equation with electron–electron scattering

In this section we solve the Boltzmann equation for n-type doped GaAs in the presence of e–e scattering of general strength. This was already done in [19] but we repeat the derivation here to keep the article self-contained. We assume that the donors, of density  $n$ , are randomly distributed and that the impurity potential can be approximated by a pure Coulomb potential. We use the values 13.0 and  $0.06 m_e$  for the dielectric constant  $\kappa$  and effective mass  $m^*$ , respectively.

The Boltzmann equation says that for the steady state condition the occupation number for a state  $k$  is unchanged with time, i.e.

$$\left. \frac{\partial f(k)}{\partial t} \right]_{\text{scatt}}^{\text{imp}} + \left. \frac{\partial f(k)}{\partial t} \right]_{\text{scatt}}^{\text{e-e}} + \left. \frac{\partial f(k)}{\partial t} \right]_{\text{field}} = 0. \quad (4)$$

The first term which comes from impurity scattering is given by

$$\left. \frac{\partial f(\mathbf{k})}{\partial t} \right]_{\text{scatt}}^{\text{imp}} = - \frac{8e^4 m^* n}{\hbar^3 K^2} \int d^3 q \frac{[f(\mathbf{k}) - f(\mathbf{k} + \mathbf{q})] \delta[q^2 + 2kq \cos(\mathbf{k}, \mathbf{q})]}{q^4 \varepsilon^2(q, 0)} \quad (5)$$

We define the transport time  $\tau(k)$  through the relation

$$f(\mathbf{k}) = f^0[\mathbf{k} + e\tau(k)E/\hbar] \quad (6)$$

where  $f^0$  is the Fermi-Dirac distribution function. Putting this expression into (5), linearizing the result and keeping the lowest order contributions leads to

$$\begin{aligned} \left. \frac{\partial f(\mathbf{k})}{\partial t} \right]_{\text{scatt}}^{\text{imp}} &= - \frac{8e^4 m^* n eE}{\hbar^3 K^2} \frac{1}{\hbar} \tau(k) \frac{\partial f^0(k)}{\partial \varepsilon_k} \frac{\partial \varepsilon_k}{\partial \mathbf{k}} \int d^3 q \frac{[1 - \cos(\mathbf{k}, \mathbf{k} + \mathbf{q})] \delta[q^2 + 2kq \cos(\mathbf{k}, \mathbf{q})]}{q^4 \varepsilon^2(q, 0)} \\ &= - \frac{eE}{\hbar} \frac{\partial f^0(k)}{\partial \varepsilon_k} \frac{\partial \varepsilon_k}{\partial \mathbf{k}} \frac{\tau(k)}{\tau_{\text{imp}}(k)} \end{aligned} \quad (7)$$

where part of the expression has been identified as the inverse transport time from impurity scattering  $1/\tau_{\text{imp}}(k)$  given by

$$1/\tau_{\text{imp}}(k) = \frac{8e^4 m^* n}{\hbar^3 K^2} \int d^3 q \frac{[1 - \cos(\mathbf{k}, \mathbf{k} + \mathbf{q})] \delta[q^2 + 2kq \cos(\mathbf{k}, \mathbf{q})]}{q^4 \varepsilon^2(q, 0)} \quad (8)$$

This is the transport time when impurity scattering is the only scattering process. The e-e scattering strives towards thermal equilibrium in the CM system. We use a relaxation time approximation for the e-e scattering and find

$$\begin{aligned} \left. \frac{\partial f(\mathbf{k})}{\partial t} \right]_{\text{scatt}}^{\text{e-e}} &= - \frac{f(\mathbf{k}) - f^0(\mathbf{k} + eE\langle\tau(k)\rangle/\hbar)}{\tau_{\text{e-e}}} \\ &= - \frac{\tau(k) - \langle\tau(k)\rangle}{\tau_{\text{e-e}}} \frac{eE}{\hbar} \frac{\partial f^0(k)}{\partial \varepsilon_k} \frac{\partial \varepsilon_k}{\partial \mathbf{k}} \end{aligned} \quad (9)$$

where the numerator on the right-hand-side of the first line is the difference between the distribution function and the Fermi-Dirac distribution function in the CM system. We have here assumed a constant value for  $\tau_{\text{e-e}}$ . This is not necessary but makes things easier. Here, we just want to estimate the importance of the e-e scattering and with this purpose it is sufficient to use an approximate relaxation time.

The last term of the Boltzmann equation is given by

$$\left. \frac{\partial f(\mathbf{k})}{\partial t} \right]_{\text{field}} = \frac{eE}{\hbar} \frac{\partial f(\mathbf{k})}{\partial \mathbf{k}} = \frac{eE}{\hbar} \frac{\partial f^0(k)}{\partial \varepsilon_k} \frac{\partial \varepsilon_k}{\partial \mathbf{k}} \quad (10)$$

Now, (4) can be written as

$$\begin{aligned} &\frac{eE}{\hbar} \frac{\partial f^0(k)}{\partial \varepsilon_k} \frac{\partial \varepsilon_k}{\partial \mathbf{k}} \frac{\tau(k)}{\tau_{\text{imp}}(k)} - \frac{eE}{\hbar} \frac{\partial f^0(k)}{\partial \varepsilon_k} \frac{\partial \varepsilon_k}{\partial \mathbf{k}} \frac{\tau(k) - \langle\tau(k)\rangle}{\tau_{\text{e-e}}} \\ &+ \frac{eE}{\hbar} \frac{\partial f^0(k)}{\partial \varepsilon_k} \frac{\partial \varepsilon_k}{\partial \mathbf{k}} = 0. \end{aligned} \quad (11)$$

After reduction and rearrangement we get

$$\tau(k) = \tau_{\text{imp}}(k)\tau_{e-e}/((\tau_{e-e} + \tau_{\text{imp}}(k)) + \langle\tau(k)\rangle\tau_{\text{imp}}(k)/(\tau_{e-e} + \tau_{\text{imp}}(k))) \quad (12)$$

and averaging both sides as prescribed in (3) results in

$$\begin{aligned} \langle\tau(k)\rangle &= \left\langle \frac{\tau_{\text{imp}}(k)\tau_{e-e}}{\tau_{e-e} + \tau_{\text{imp}}(k)} \right\rangle / \left( 1 - \left\langle \frac{\tau_{\text{imp}}(k)}{\tau_{e-e} + \tau_{\text{imp}}(k)} \right\rangle \right) \\ &= \left\langle \frac{\tau_{e-e}}{\tau_{e-e}/\tau_{\text{imp}}(k) + 1} \right\rangle / \left\langle \frac{\tau_{e-e}/\tau_{\text{imp}}(k)}{\tau_{e-e}/\tau_{\text{imp}}(k) + 1} \right\rangle. \end{aligned} \quad (13)$$

Here we see that for vanishing e-e scattering  $\langle\tau(k)\rangle = \langle\tau_{\text{imp}}(k)\rangle$ , i.e., result number one is obtained. On the other hand very strong e-e scattering gives  $\langle\tau(k)\rangle = 1/(1/\tau_{\text{imp}}(k))$  and result number two is regained.

### 3. Relaxation time for electron-electron scattering

In this section we use a trick to lure the GDA formalism to give us the value of the e-e relaxation time. As we have mentioned before the e-e scattering gives no direct contribution to the resistivity in a single-component plasma. The reason for this is not the absence of e-e scattering. The reason is that both momentum and current are preserved in each e-e scattering event. In a two-component plasma only momentum is conserved, and the scattering contributes directly to the resistivity. The contribution comes in the form of the scattering rate times a prefactor. This prefactor vanishes in the single-component plasma but not for the two-component plasma if the charge-over-mass ratios are different for the two components. This means that the scattering rate can be extracted from the theory even for a single-component plasma, if proper care is taken.

To reach our goal we add to our electron system a 'shadow system' with the same number of carriers as electrons but with a different mass; we calculate the scattering rate between the components in this combined system, let the masses become equal and in the end we eliminate the 'shadow system'. We use the GDA [15] to obtain the scattering rates. This approach consists of three steps. In the first step the high frequency limit of the dynamical conductivity is derived within the Kubo formalism and diagrammatic perturbation theory. In the second step, this result is compared to the high-frequency expansion of the generalised Drude expression for the dynamical conductivity and the various relaxation times are hereby identified (the generalisation of the Drude expression consists of allowing the relaxation times to be frequency dependent and complex valued). In the third, and last, step the obtained expression for the relaxation times are assumed to be valid for all frequencies and in particular for zero frequency.

Let the indices 1 and 2 represent the real carriers and those of the 'shadow system', respectively. The averaging velocities of the carriers are denoted by  $v$ . Since at the end we will let the masses become equal we can from the start let the coefficients of friction between the carriers and the impurities be the same,  $\eta$ , for the two groups of carriers. The coefficient of mutual friction we denote by  $\eta_{12}$ . With these definitions the equations of motion in an applied electric field  $E$  become

$$\begin{aligned} m_1 v_1(-i\omega) &= eE - n v_1 \eta - n(v_1 - v_2)\eta_{12} \\ m_2 v_2(-i\omega) &= eE - n v_2 \eta + n(v_1 - v_2)\eta_{12}. \end{aligned} \quad (14)$$

The coefficient of mutual friction is related to the relaxation time for scattering between

the carriers. This relation is obtained from realising that as the electric field and the impurity scattering are turned off the relative velocity between the two groups of carriers decays with the decay constant  $1/\tau_{12}$ . For vanishing  $E$  and  $\eta$  (14) reduces, after the inverse Fourier transform has been taken, into

$$dv_1/dt = -\eta_{12}(n/m_1)(v_1 - v_2) \quad dy_2/dt = \eta_{12}(n/m_2)(v_1 - v_2). \tag{15}$$

Subtraction of the two equations leads to the following time dependence of the relative velocity

$$(v_1 - v_2) \propto \exp[-\eta_{12}n(1/m_1 + 1/m_2)t] \tag{16}$$

and  $\tau_{12}$  can be identified as

$$\tau_{12}^{-1} = \eta_{12}n(1/m_1 + 1/m_2). \tag{17}$$

Solving (14) and noting that the conductivity,  $\sigma$ , can be expressed as

$$\sigma = (ne/E)(v_1 + v_2) \tag{18}$$

we arrive at the following expression for the dynamical conductivity

$$\sigma = -ne^2 \times \frac{[2n\eta + 4n\eta_{12} - i(m_1 + m_2)\omega]}{[1 - (n\eta)^2 - (4n\eta_{12})^2 - 2n^2\eta\eta_{12} + m_1m_2\omega^2 + i\omega(m_1 + m_2)(n\eta + n\eta_{12})]}. \tag{19}$$

The high-frequency expansion of this expression is

$$\sigma \underset{\omega \rightarrow \infty}{\approx} (ne^2/\omega^2)[n\eta(1/m_1^2 + 1/m_2^2)] + (ne^2/\omega^2)[n\eta_{12}(1/m_1 - 1/m_2)^2] \tag{20}$$

where the first term is the contribution from impurity scattering and the second is the contribution from e-e scattering. We see that the expression behaves correctly in that the last contribution vanishes for equal masses.

The contribution from e-e scattering is compared to the result for a two-component plasma derived within the Kubo formalism in [24] (equation (2.30) of that reference). This expression also contains the same prefactor as the second term of (20). The coefficient of mutual friction is identified and the static limit is taken. The result is

$$\eta_{12} = \frac{\hbar^2\beta}{n^2} \int_0^\infty \frac{d\omega}{2\pi} \frac{1}{\sinh^2\hbar\beta\omega/2} \int \frac{dq}{(2\pi)^3} q_\mu^2 \frac{\text{Im } \alpha_1(\mathbf{q}, \omega)\text{Im } \alpha_2(\mathbf{q}, \omega)}{|\epsilon(\mathbf{q}, \omega)|^2} \tag{21}$$

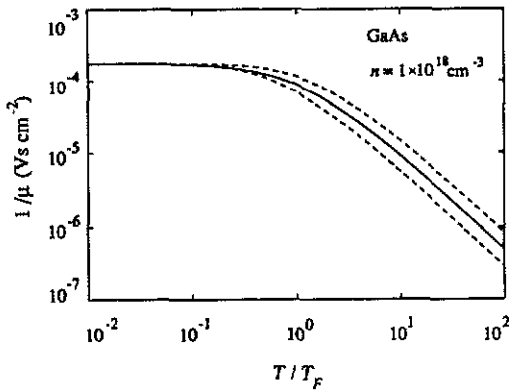
where  $\alpha_1$  and  $\alpha_2$  are the polarisabilities for the two carrier systems and  $\epsilon$  is the dielectric function

$$\epsilon(\mathbf{q}, \omega) = 1 + \alpha_1(\mathbf{q}, \omega) + \alpha_2(\mathbf{q}, \omega). \tag{22}$$

The quantity  $q_\mu$  is the projection of the momentum on the electric field.

Now we can safely let  $m_1 = m_2 = m^*$  and in doing so we should only keep one of the polarisabilities in the dielectric function. Thus, the coefficient of friction is

$$\eta_{e-e} = \frac{\hbar^2\beta}{n^2} \int_0^\infty \frac{d\omega}{2\pi} \frac{1}{\sinh^2\hbar\beta\omega/2} \int \frac{dq}{(2\pi)^3} q_\mu^2 \frac{\text{Im}^2 \alpha_0(\mathbf{q}, \omega)}{|1 + \alpha_0(\mathbf{q}, \omega)|^2} \tag{23}$$



**Figure 1.** The reciprocal mobility as function of  $T/T_F$  for the donor density  $n = 1 \times 10^{18} \text{ cm}^{-3}$ . The lower and upper broken curve (----) are results number one and two, respectively, as defined in the text. The solid curve (—) is our full result, including e-e scattering.

and the e-e scattering rate is finally obtained as

$$1/\tau_{e-e} = \frac{2\hbar^2 \beta}{m^* n} \int_0^\infty \frac{d\omega}{2\pi \sinh^2 \hbar\beta\omega/2} \int \frac{dq}{(2\pi)^3} q_\mu^2 \frac{\text{Im}^2 \alpha_0(q, \omega)}{|1 + \alpha_0(q, \omega)|^2}. \quad (24)$$

The polarisabilities are the full temperature dependent polarisabilities in the random phase approximation (RPA).

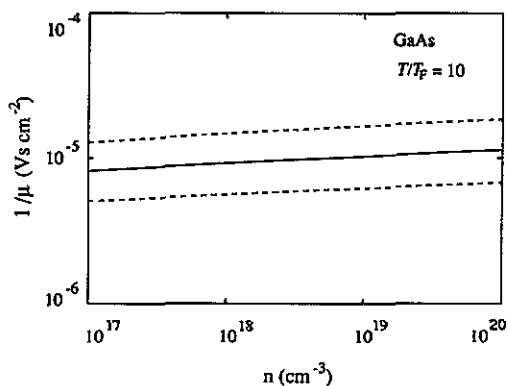
This completes the derivation of the relaxation time for e-e scattering. In the next section we present the numerical results.

#### 4. Numerical results

In this section we present our numerical results for doped GaAs. These results are the solution of the Boltzmann equation for the impurity scattering in the presence of electron-electron scattering. To tie up with our previous article [19] we present the results in the form of the reversal mobility:

$$1/\mu = ne\rho = ne/\sigma = m^*/e\langle\tau(k)\rangle. \quad (25)$$

The expression for  $\langle\tau(k)\rangle$  was given in (13). As we already stated in section 2 we assume that the donors, of density  $n$ , are randomly distributed and that the impurity potential can be approximated by a pure Coulomb potential. We use the values 13.0 and  $0.06 m_e$  for the dielectric constant  $\kappa$  and effective mass  $m^*$ , respectively. In figure 1 we give the result for  $n = 1 \times 10^{18} \text{ cm}^{-3}$  as a function of  $T/T_F$ . The lower chained curve is result number one and the upper chained curve is result number two. The solid curve is our full result from the Boltzmann equation with e-e scattering included. As can be seen this curve is close to result number one for lower temperatures. Then it separates from result number one and for higher temperatures stays roughly at equal distance from results number one and two (note the logarithmic scale). To investigate if there is a systematic density variation in the relative importance of the e-e scattering we made a calculation in which  $T/T_F$  was kept constant at the value 10 and the density varied over a region covering a variation of three orders of magnitude. We found that the behaviour was very much the same over the whole density range. These results are presented in



**Figure 2.** The reciprocal mobility as function of donor density for constant  $T/T_F = 10$ . The lower and upper broken curves (-----) are results number one and two, respectively, as defined in the text. The solid curve (—) is our full result, including e-e scattering.

figure 2. Here, again the lower curve is result number one and the uppermost curve result number two. The full result is given by the solid curve which lies more or less in the middle. We should remember that the dashed curves are the two extreme results with no and very strong e-e scattering, respectively. All results were obtained with full temperature-dependent RPA screening.

## 5. Summary and conclusions

We have studied the indirect effects that e-e scattering has on the impurity resistivity. Numerical results have been presented for doped GaAs. All important theories of impurity resistivity give, in the absence of e-e scattering, one of two results. It turns out that these two results are the extreme limits of the results from a more elaborate calculation including e-e scattering. In the limit of very strong e-e scattering the electrons are thermalised in their centre-of-mass system; in this system the distribution function for the electrons is a Fermi-Dirac distribution function. In the other extreme limit, i.e. in the limit of no e-e scattering at all, the distribution function is different. The impurity scattering depends on the form of the distribution function, which means that the e-e scattering has an indirect effect on the resistivity. In formulations where a basic assumption is that the electrons are thermalised in their centre-of-mass system the result is the extreme limit with very strong e-e scattering even though no explicit e-e scattering is included in the formalism.

We used the Boltzmann equation including impurity and e-e scattering to calculate the reversal mobility. The impurity part was included in a strict fashion while the e-e scattering part was treated in a relaxation time approximation. The e-e scattering-rate was obtained from the generalised Drude approach. We found that the inclusion of the actual e-e scattering in doped GaAs led to a result roughly in the middle of the two extreme results. Thus, the e-e scattering was neither negligible nor strong enough to thermalise the electrons in the centre-of-mass system. This behaviour was independent of the doping level.

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